Practice Exercises

Partial Fractions

1. Express the following in partial fractions;

   (a) $3\,\dfrac{1}{(x+1)\,(x-1)}$ (c) $\dfrac{x-1}{(x+2)\,(x-2)}$ (e) $\dfrac{x+3}{x\,(x+1)}$

   (b) $\dfrac{x}{(x-4)\,(x-1)}$ (d) $2\,\dfrac{1}{(2\,x-1)\,(x-2)}$ (f) $\dfrac{2\,x-1}{(x+1)\,(3\,x+2)}$

2. (a) $3\,\dfrac{x}{(x-1)\,(x-2)\,(x-3)}$ (b) $\dfrac{x^2-2\,x+4}{x\,(x-3)\,(x+1)}$

3. (a) $2\,\dfrac{1}{(x-1)\,(x^2+1)}$ (c) $\dfrac{x+3}{x\,(x^2+2)}$ (e) $\dfrac{x^2+1}{x\,(2\,x^2-1)\,(x-1)}$

   (b) $\dfrac{x-3}{(x+4)\,(x^2-2)}$ (d) $\dfrac{2\,x^2+x+1}{(x-3)\,(2\,x^2-1)}$ (f) $\dfrac{2\,x^2+1}{(x^2+1)\,(x-1)}$

4. Convert the following expression to partial fractions.

$$\frac{x^3-1}{(x+2)\,(1+2\,x)\,(x^2+1)}$$

5. By factorising the denominator write the following expression in partial fractions.

$$\frac{x^2+x+1}{(x^2-1)\,(x^2+1)}$$

6. Express the following in partial fractions;

   (a) $\dfrac{x}{(x-1)\,(x-2)^2}$ (c) $3\,\dfrac{1}{x\,(3\,x-1)^2}$ (e) $\dfrac{1}{(x-2)\,(x+3)^2}$

   (b) $\dfrac{x^2-1}{x^2\,(2\,x+2)}$ (d) $\dfrac{1}{(x+2)\,(x-1)^2}$ (f) $\dfrac{x-2}{(3\,x+1)\,(x-1)^2}$

7. Given that

$$\frac{x^2-2}{(x+3)\,(x-1)} = A + \frac{B}{x+3} + \frac{C}{x-1}$$

   find the values of the constants A, B and C.

8. Given that

$$\frac{x^2-x-24}{(x+2)\,(x-4)} = A + \frac{B}{x+2} + \frac{C}{x-4}$$

   find the values of the constants A, B and C.

Answers

1. (a) $-3/2\ (x+1)^{-1} + 3/2\ (x-1)^{-1}$
   (b) $4/3\ (x-4)^{-1} - 1/3\ (x-1)^{-1}$
   (c) $3/4\ (x+2)^{-1} + 1/4\ (x-2)^{-1}$
   (d) $-4/3\ (2\,x-1)^{-1} + 2/3\ (x-2)^{-1}$
   (e) $3\,x^{-1} - 2\ (x+1)^{-1}$
   (f) $3\ (x+1)^{-1} - 7\ (3\,x+2)^{-1}$

2. (a) $3/2\ (x-1)^{-1} - 6\ (x-2)^{-1} + 9/2\ (x-3)^{-1}$
   (b) $-4/3\,x^{-1} + \dfrac{7}{12}\ (x-3)^{-1} + 7/4\ (x+1)^{-1}$

3. (a) $(x-1)^{-1} - \dfrac{x+1}{x^2+1}$
   (b) $-1/2\ (x+4)^{-1} + 1/2\,\dfrac{x-2}{x^2-2}$
   (c) $3/2\,x^{-1} - 1/2\,\dfrac{-2+3\,x}{x^2+2}$
   (d) $\dfrac{22}{17}\ (x-3)^{-1} - 1/17\,\dfrac{13+10\,x}{2\,x^2-1}$
   (e) $x^{-1} + 2\ (x-1)^{-1} - 3\,\dfrac{1+2\,x}{2\,x^2-1}$
   (f) $3/2\ (x-1)^{-1} + 1/2\,\dfrac{x+1}{x^2+1}$

4. $3/5\ (x+2)^{-1} - 3/5\ (1+2\,x)^{-1} + 1/5\,\dfrac{x-1}{x^2+1}$

5. $3/4\ (x-1)^{-1} - 1/4\ (x+1)^{-1} - 1/2\,\dfrac{x}{x^2+1}$

6. (a) $(x-1)^{-1} + 2\ (x-2)^{-2} - (x-2)^{-1}$
   (b) $-1/2\,x^{-2} + 1/2\,x^{-1}$
   (c) $3\,x^{-1} + 9\ (3\,x-1)^{-2} - 9\ (3\,x-1)^{-1}$
   (d) $1/9\ (x+2)^{-1} + 1/3\ (x-1)^{-2} - 1/9\ (x-1)^{-1}$
   (e) $1/25\ (x-2)^{-1} - 1/5\ (x+3)^{-2} - 1/25\ (x+3)^{-1}$
   (f) $-\dfrac{21}{16}\ (3\,x+1)^{-1} - 1/4\ (x-1)^{-2} + \dfrac{7}{16}\ (x-1)^{-1}$

7. $1 - 7/4\ (x+3)^{-1} - 1/4\ (x-1)^{-1}$

8. $1 + 3\ (x+2)^{-1} - 2\ (x-4)^{-1}$

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